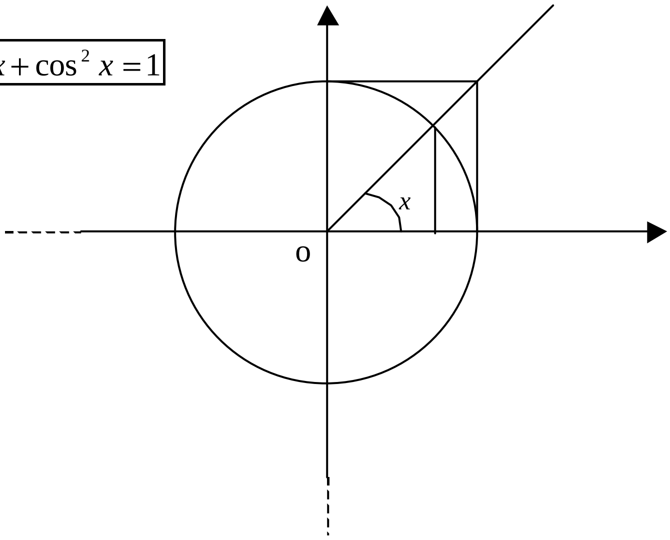
FORMULE TRIGONOMETRICHE

$$\operatorname{sen}^2 x + \cos^2 x = 1$$



sen x e cos x come funzioni razionali di $\operatorname{tg}\frac{x}{2}$:

$$\operatorname{sen} x = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}; \qquad \cos x = \frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}.$$

Formule di ***addizione*** e ***sottrazione***:

$$\operatorname{sen}(a+b) = \operatorname{sen} a \cos b + \cos a \operatorname{sen} b; \qquad \operatorname{sen}(a-b) = \operatorname{sen} a \cos b - \cos a \operatorname{sen} b;$$

$$\cos(a+b) = \cos a \cos b - \operatorname{sen} a \operatorname{sen} b; \qquad \cos(a-b) = \cos a \cos b + \operatorname{sen} a \operatorname{sen} b;$$

$$\operatorname{tg}(a+b) = \frac{\operatorname{tg} a + \operatorname{tg} b}{1-\operatorname{tg} a \operatorname{tg} b}; \qquad \operatorname{tg}(a-b) = \frac{\operatorname{tg} a - \operatorname{tg} b}{1+\operatorname{tg} a \operatorname{tg} b}.$$

Formule di ***Prostaferesi***:

$$\operatorname{sen} a + \operatorname{sen} b = 2\operatorname{sen}\frac{a+b}{2}\cos\frac{a-b}{2}; \qquad \operatorname{sen} a - \operatorname{sen} b = 2\cos\frac{a+b}{2}\operatorname{sen}\frac{a-b}{2};$$

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}; \qquad \cos a - \cos b = -2\operatorname{sen}\frac{a+b}{2}\operatorname{sen}\frac{a-b}{2};$$

$$\operatorname{tg} a + \operatorname{tg} b = \frac{\operatorname{sen}(a+b)}{\cos a \cos b}; \qquad \operatorname{tg} a - \operatorname{tg} b = \frac{\operatorname{sen}(a-b)}{\cos a \cos b}. \qquad ?$$

Formule di ***Werner***:

$$\operatorname{sen} a \operatorname{sen} b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right];$$

$$\operatorname{sen} a \cos b = \frac{1}{2}\left[\operatorname{sen}(a+b) + \operatorname{sen}(a-b)\right];$$

$$\cos a \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right].$$

Formule di ***duplicazione***:

$$\operatorname{sen} 2x = 2\operatorname{sen} x \cos x; \qquad \cos 2x = \cos^2 x - \operatorname{sen}^2 x; \qquad \operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x}.$$

Formule di ***bisezione***:

$$\operatorname{sen}\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}\,; \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}\,; \qquad tg\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}\,.$$

Formule di ***Eulero***:

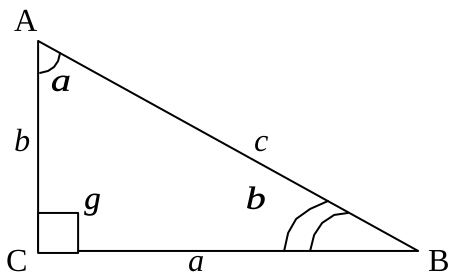
$$e^{ix} = \cos x + i \operatorname{sen} x\,; \qquad e^{-ix} = \cos x - i \operatorname{sen} x\,;$$

$$\operatorname{sen} x = \frac{e^{ix} - e^{-ix}}{2i}\,; \qquad \cos x = \frac{e^{ix} + e^{-ix}}{2}\,.$$

Quadrato di seno e coseno:

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}\,; \qquad \cos^2 x = \frac{1+\cos 2x}{2}\,.$$

In un *triangolo rettangolo*:



$$\operatorname{sen}\boldsymbol{a} = \frac{a}{c};\ \operatorname{sen}\boldsymbol{b} = \frac{b}{c};\ \cos\boldsymbol{a} = \frac{b}{c};\ \cos\boldsymbol{b} = \frac{a}{c};$$

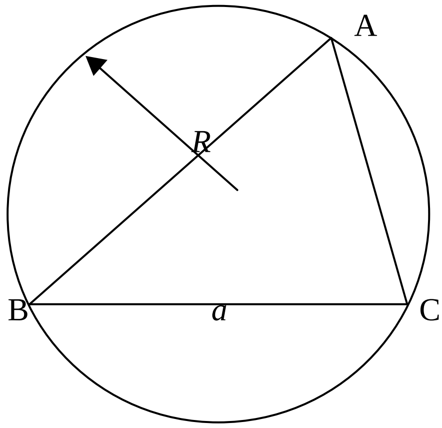
$$\operatorname{tg}\boldsymbol{a} = \frac{a}{b};\ \operatorname{tg}\boldsymbol{b} = \frac{b}{a};\ \operatorname{cotg}\boldsymbol{a} = \frac{b}{a};\ \operatorname{cotg}\boldsymbol{b}\frac{a}{b};$$

$$\sec\boldsymbol{a}\frac{c}{b};\ \sec\boldsymbol{b} = \frac{c}{a};\ \operatorname{cosec}\boldsymbol{a} = \frac{c}{a};\ \operatorname{cosec}\boldsymbol{b} = \frac{c}{b};$$

$$b = c\operatorname{sen}\boldsymbol{b};\ b = c\cos\boldsymbol{a};\ b = a\operatorname{tg}\boldsymbol{b};\ b = a\operatorname{cotg}\boldsymbol{a};$$

$$a = c\operatorname{sen}\boldsymbol{a};\ a = c\cos\boldsymbol{b};\ a = b\operatorname{tg}\boldsymbol{a};\ c = b\operatorname{cotg}\boldsymbol{b};$$

$$c = \frac{a}{\operatorname{sen}\boldsymbol{a}};\ c = \frac{a}{\cos\boldsymbol{b}};\ c = \frac{b}{\operatorname{sen}\boldsymbol{b}};\ c = \frac{b}{\cos\boldsymbol{a}}.$$



In tutti i triangoli:

$\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{g}=180°$

Teorema *dei seni*:

$$\frac{a}{\text{sen}\,\boldsymbol{a}}=\frac{b}{\text{sen}\,\boldsymbol{b}}=\frac{c}{\text{sen}\,\boldsymbol{g}}=2R$$

dove R è il raggio della circonferenza circoscritta al triangolo

Teorema di ***Carnot***:

$a^2=b^2+c^2-2bc\cos\boldsymbol{a}$; $b^2=a^2+c^2-2ac\cos\boldsymbol{b}$; $c^2=a^2+b^2-2ab\cos\boldsymbol{g}$.

Teorema di ***Nepero***:

$$\frac{a-b}{a+b}=\frac{\text{tg}\,\frac{\boldsymbol{a}-\boldsymbol{b}}{2}}{\text{tg}\,\frac{\boldsymbol{a}+\boldsymbol{b}}{2}};\qquad \frac{b-c}{b+c}=\frac{\text{tg}\,\frac{\boldsymbol{b}-\boldsymbol{g}}{2}}{\text{tg}\,\frac{\boldsymbol{b}+\boldsymbol{g}}{2}};\qquad \frac{c-a}{c+a}=\frac{\text{tg}\,\frac{\boldsymbol{g}-\boldsymbol{a}}{2}}{\text{tg}\,\frac{\boldsymbol{g}+\boldsymbol{a}}{2}}.$$

Teorema *delle proiezioni*:

$a=b\cos\boldsymbol{g}+c\cos\boldsymbol{b}$; $b=c\cos\boldsymbol{a}+a\cos\boldsymbol{g}$; $c=a\cos\boldsymbol{b}+b\cos\boldsymbol{a}$.

Area dei triangoli:

$A=\frac{1}{2}ab\,\text{sen}\,\boldsymbol{g}=\frac{1}{2}bc\,\text{sen}\,\boldsymbol{a}=\frac{1}{2}ac\,\text{sen}\,\boldsymbol{b}$ (conoscendo due lati ed un angolo)

$A=\sqrt{p(p-a)(p-b)(p-c)}$ Formula di Erone (conoscendo tre lati)
p è il semiperimetro

$A=\frac{a^2\,\text{sen}\,\boldsymbol{b}\,\text{sen}\,\boldsymbol{g}}{2\,\text{sen}\,\boldsymbol{a}}=\frac{b^2\,\text{sen}\,\boldsymbol{g}\,\text{sen}\,\boldsymbol{a}}{2\,\text{sen}\,\boldsymbol{b}}=\frac{c^2\,\text{sen}\,\boldsymbol{a}\,\text{sen}\,\boldsymbol{b}}{2\,\text{sen}\,\boldsymbol{g}}$ (conoscendo tre angoli ed un lato)

$A=r\cdot p$, con r raggio della circonferenza iscritta e p semiperimetro

$A=\frac{abc}{4R}$, con R raggio della circonferenza circoscritta e p semiperimetro